**1. Proof that the product of any even integer and any other integer is even. (proof by direct)**

Let x = 2m (the even integer), let y = 2n (the second even integer), and let z = 2p + 1 (the odd integer), for some integer m, n, and p, respectively.

Proof that the product of any two even integers is even:

1. xy = 2m \* 2n
2. xy = 4mn

↳ QED

Proof that the product of any even integer and any odd integer is even:

1. xz = (2m)(2p + 1)
2. xz = 2mp + 2m
3. xz = 2m(p + 1)

↳ QED

**2. For n a positive integer, if n > 2 and n is prime then n is odd. (proof by contradiction)**

The contradiction is ㄱ(n is odd), meaning n is even. Now we have to prove ([n is >2 and prime] ∧ ㄱ[n is odd] ⟶ False), or in other words, prove that the theorem, “for any prime integer n that is greater than 2, n must be even”, is false.

The only even prime number is 2, and since it’s given that n must be greater than 2, we just proved the theorem is false. Therefore, if n is greater than 2 and is prime, n must be odd.

**3. If x, y are integers and x and y are both odd, then x + y is even. Prove using the following schemes. Consider all cases.**

1. **Direct Proof**

Because x and y are odd integers, let x = 2m + 1 and let y = 2n + 1 for some integer m and n, respectively.

1. x + y = (2m + 1) + (2n + 1)
2. x + y = 2m + 2n + 2
3. x + y = 2(m + n + 1)
4. Let i = m + n + 1
5. ∴ x + y = 2(i), which by definition is an even number.

Thus, the theorem asked in the question is true.

1. **Proof by Contrapositive**

To start, the contrapositive of the statement is:

ㄱ(x + y is even) ⟶ ㄱ(either x or y is odd or even).

To prove this theorem, we need to prove either x or y is odd or even.

Suppose x is odd, and given x + y is odd due to the contrapositive theorem:

↳ (x + y) - x has an odd solution.

∴ y is even.

Suppose y is even, and given x + y is odd:

↳ (x + y) - y has an even solution.

∴ x is odd.

↳ QED

1. **Proof by Contradiction**

The contradiction is ㄱ(x + y), meaning x + y yields an odd sum.

We now need to prove ([x and y are odd] ∧ ㄱ[x + y] ⟶ False), or in other words, prove that the theorem, “because x and y are odd integers, the sum of x and y is odd”, is false.

Looking back at part a., where we proved that the sum of any two odd numbers is even, we can say the theorem above is false. Thus, the theorem we are asked to prove in the question is true.

**4. Decide which one of the following statements is an example of Inductive OR Deductive reasoning:**

1. **It is dangerous to drive on icy streets. The streets are icy now, so it is dangerous to drive now.**

Deductive

1. **Elephants have cells in their bodies and all cells have DNA, so elephants have DNA.**

Deductive

1. **All birds have feathers and robins are birds, so robins have feathers.**

Deductive

1. **Red meat has iron in it and beef is red meat, so beef has iron in it.**

Deductive

1. **John is an excellent swimmer. John’s family has a swimming pool. John’s sister Mary must also be an excellent swimmer.**

Inductive

1. **Ray is a football player. All football players weigh more than 170 pounds. Ray weighs more than 170 pounds.**

Deductive

**5. Prove by induction that for every positive integer n:**

1. , is TRUE
2. , is TRUE
3. is TRUE.

With induction, assume is true when n = j, j being some positive integer.

We need to prove that is true, given the induction hypothesis.

↳ QED

**6. Prove by induction that the sum of the first n positive even integers is .**

We need to show that (n + 1) is true, given the induction hypothesis.

1. 2 + 4 + 6 + … + 2n =
2. Add 2n + 2 on both sides:

2 + 4 + 6 + … + 2n + (2n +2) = + (2n + 2)

1. Taking the right hand side:

+ (2n + 2) = = =

With this, we prove that both n and (n + 1) holds true under the hypothesis.